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Solutions to Written Exam at the Department of Economics summer 2020

Economics of Exchange Rates

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Number of questions: This exam consists of 2 questions.

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1. **Portfolio Balance Model**

Consider the standard Portfolio Balance Model comprised of the following functions

$$W \equiv M + B_p + SF_p \tag{1}$$

$$M = m\left(r, E\dot{s}, Y, W\right) \quad m_r < 0, m_{E\dot{s}} < 0, m_y > 0, m_w > 0 \tag{2}$$

$$B_p = b\left(r, E\dot{s}, Y, W\right) \quad b_r > 0, b_{E\dot{s}} < 0, b_y < 0, b_w > 0 \tag{3}$$

$$SF_p = f\left(r, E\dot{s}, Y, W\right) \quad f_r < 0, f_{E\dot{s}} > 0, f_y < 0, f_w > 0 \tag{4}$$

Notation is standard.

(a) Consider first the short-term version of this model, i.e., we assume that prices are constant. Given the formal model above, explain what happens to (a) the interest rate that maintains money market equilibrium following a rise in the exchange rate, (b) the interest rate that maintains bond market equilibrium following a rise in the exchange rate, and (c) the exchange rate that maintains equilibrium in the market for foreign bonds following a rise in the interest rate.

(b) We can solve the portfolio balance model given the assumptions that $dY = dE\dot{s} = 0$ to find that

$$\begin{bmatrix} dr \\ dW \\ dS \end{bmatrix} = \begin{bmatrix} \frac{m_w}{b_r m_w - b_w m_r} & 0 & -\frac{b_w}{b_r m_w - b_w m_r} \\ -\frac{m_r}{b_r m_w - b_w m_r} & 0 & \frac{b_r}{b_r m_w - b_w m_r} \\ \frac{f_r m_w - f_w m_r}{F_p(b_r m_w - b_w m_r)} & -\frac{1}{F_p} & \frac{b_r f_w - b_w f_r}{F_p(b_r m_w - b_w m_r)} \end{bmatrix} \begin{bmatrix} dB_p \\ SdF_p \\ dM \end{bmatrix}$$

Assume that the government has decided to stimulate the economy using fiscal policy. Use the model above to derive the analytical effects of a money financed expansionary fiscal policy. Explain carefully.

(c) An alternative to money financed fiscal stimulus is to use bond financing. Use the model above to derive the analytical effects of a bond financed fiscal stimulus. Explain carefully.

(d) Relax the assumption that the price level is constant. We still assume a small-open economy and that the current account is in balance, and net exports is zero initially. Assume that the monetary authority expands the money stock by purchasing domestic assets. Show how the expansionary monetary policy affects prices, net exports and the exchange rate in the long-run. A graphical and verbal analysis suffice. [Hint: Assume that the economy is in full equilibrium initially, the three asset markets are in equilibrium, net foreign investment income is zero and the trade balance is zero such that the current account is zero. Normalize the exchange rate and the price level such that they are both equal to unity.]

2. **Micro-based macro model**

Consider the following version of the micro-based macro model by Evans. It is assumed that the log spot price quoted by dealers at the start of time period t is given by UIP with a risk premium

$$s_t = \mathbb{E}_t^D s_{t+1} + \hat{r}_t - r_t - \delta_t. \tag{5}$$

where s_t is the log of the spot exchange rate measured as home currency units per unit foreign currency, r_t is the home interest rate, $\hat{r}_t$ is the foreign interest rate and δ_t is the risk premium. We will assume that the home country sets the interest rate using an extended Taylor-rule where the interest rate is set conditional on a real exchange rate target. Dealers then expect that the domestic interest rate at time $t+i$ is set according to

$$\mathbb{E}_t^D r_{t+i} = \gamma_\pi \mathbb{E}_t^D \Delta p_{t+1+i} + \gamma_y \mathbb{E}^D y_{t+i} + \gamma_\varepsilon \mathbb{E}_t^D \varepsilon_{t+i} \tag{6}$$

where the real exchange rate ε_t is defined as

$$\varepsilon_t \equiv s_t + \hat{p}_t - p_t. \tag{7}$$

whereas dealers expect that the foreign interest rate is set according to a conventional Taylor-rule

$$\mathbb{E}_t^D \hat{r}_{t+i} = \gamma_\pi \mathbb{E}_t^D \Delta \hat{p}_{t+1+i} + \gamma_y \mathbb{E}^D \hat{y}_{t+i} \tag{8}$$

(a) Derive the dealers' expectation of the spot exchange rate in period $t+1$ conditional on public information available at time t, i.e., show that the dealers' expectation has the following form

$$\mathbb{E}_t^D s_{t+1} = \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i (f_{t+i} - \delta_{t+i}) \tag{9}$$

for suitable definitions of the fundamentals f_t and the discount factor ρ.

(b) Show that the equilibrium spot exchange rate at time t can be written as

$$s_t = (\hat{r}_t - r_t) + \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i f_{t+i} - \mathbb{E}_t^D \sum_{i=0}^{\infty} \rho^i \delta_{t+i} \tag{10}$$

(c) The micro-based macro model has implications for how new information, for example news about monetary policy, oral interventions and other communication from policy makers affect exchange rates. President Trump is very active on twitter commenting on various issues. The following was posted on August 30, 2019 at 3:55PM: "The Euro is dropping against the Dollar "like crazy," giving them a big export and manufacturing advantage...and the Fed does NOTHING! Our Dollar is now the strongest in history. Sounds good, doesn't it? Except to those (manufacturers) that make product for sale outside the U.S." Use the model above to explain how you think this statement should affect the USD/Euro exchange rate in the short-run as well as in the long-run.

Solutions:

1. This question relates to the following learning objectives. Knowledge: Describe the main models of exchange rate determination. Skills: Master and apply the main models of exchange rate determination to analyze the effects of monetary and fiscal policy on the exchange rate.

   The question focuses on the portfolio balance model given by equations (1) to (4) where the signs of the partial derivatives of the three demand functions are also shown.

   (a) In the first question we are asked to describe what happens to the interest rate or the exchange rate when one of the three markets is in equilibrium. The question directly relates to the slope of the equilibrium curves in a graphical representation of the model.

   (a) If the money market is in equilibrium, for a constant interest rate and a rise in the exchange rate (a depreciation) then the home currency value of foreign assets held by domestic households rises. This raises wealth who respond to this increase in wealth by rebalance their portfolios. This creates excess demand for money which can only be offset by an increase in the interest rate given the existing money stock. The money market equilibrium curve must then be upward-sloping in the interest rate-exchange rate plane.

   (b) In equilibrium, the bond market equilibrium curve must be downward-sloping such that at the equilibrium interest rate, a rise in the exchange rate implies that the home currency value of foreign bonds increase and investors rebalance their portfolios. Excess demand for domestic bonds increase the price on bonds and therefore the interest rate must fall.

   (c) If the market for foreign bonds is in equilibrium, a rise in the interest rate reduces the price on domestic bonds making them more attractive. Investors then sell foreign bonds and buy domestic bonds. Excess supply of foreign bonds requires a fall in the exchange rate. The equilibrium curve must then be downward-sloping in the interest rate-exchange rate plane.

   (b) Next we focus on the effects of fiscal policy. There in principle two ways to finance increased budget deficits, either money financing or by borrowing from the public. In the first case, the money stock increases by the amount of the deficit (which in turn is equal to the increase in wealth). In the second case, public holdings of domestic bonds is equal to the deficit. Let D be the deficit, then using money financing, $dM = D$. Using the solution of the model given in the question we then have

$$\begin{bmatrix} dr \\ dW \\ dS \end{bmatrix} = \begin{bmatrix} \frac{m_w}{b_r m_w - b_w m_r} & 0 & -\frac{b_w}{b_r m_w - b_w m_r} \\ -\frac{m_r}{b_r m_w - b_w m_r} & 0 & \frac{b_r}{b_r m_w - b_w m_r} \\ \frac{f_r m_w - f_w m_r}{F_p(b_r m_w - b_w m_r)} & -\frac{1}{F_p} & \frac{b_r f_w - b_w f_r}{F_p(b_r m_w - b_w m_r)} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ D \end{bmatrix}$$

   implying that

$$dr = -\frac{b_w D}{b_r m_w - b_w m_r} < 0$$

since $b_w > 0$ and $b_r m_w - b_w m_r > 0$ since $m_r < 0$ whereas b_r, b_w, and m_w are all positive.

$$dW = \frac{b_r D}{b_r m_w - b_w m_r} > 0$$

and

$$dS = \frac{b_r f_w - b_w f_r}{F_p(b_r m_w - b_w m_r)} D > 0$$

since $f_r < 0$ and $f_w > 0$. The interest rate falls, wealth increases and the exchange rate increases (depreciates).

(c) Bond financing implies that $dB = D$ such that the solution can be written as

$$\begin{bmatrix} dr \\ dW \\ dS \end{bmatrix} = \begin{bmatrix} \frac{m_w}{b_r m_w - b_w m_r} & 0 & -\frac{b_w}{b_r m_w - b_w m_r} \\ -\frac{m_r}{b_r m_w - b_w m_r} & 0 & \frac{b_r}{b_r m_w - b_w m_r} \\ \frac{f_r m_w - f_w m_r}{F_p(b_r m_w - b_w m_r)} & -\frac{1}{F_p} & \frac{b_r f_w - b_w f_r}{F_p(b_r m_w - b_w m_r)} \end{bmatrix} \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix}$$

implying that

$$dr = \frac{m_w}{b_r m_w - b_w m_r} D > 0$$

$$dW = -\frac{m_r}{b_r m_w - b_w m_r} > 0$$

since $m_r < 0$ and

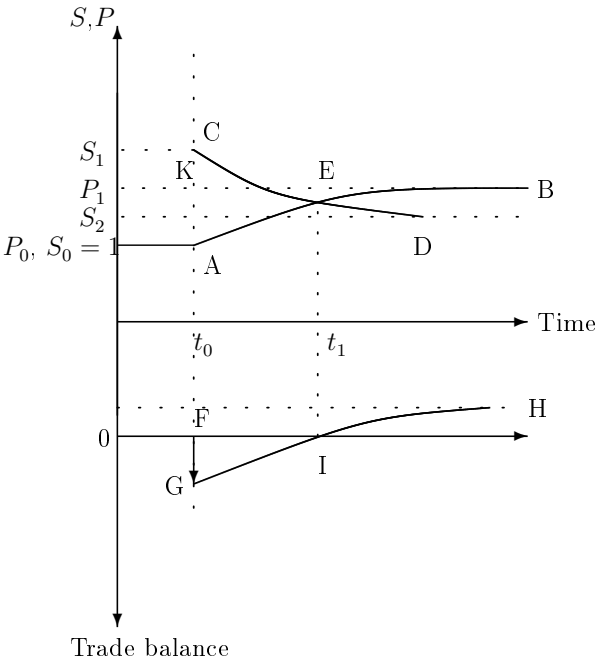
$$dS = \frac{f_r m_w - f_w m_r}{F_p(b_r m_w - b_w m_r)} D \gtrless 0$$

We find unambiguously that the interest rate increases and wealth increases but the effect on the exchange rate is ambiguous, either a depreciation or an appreciation. An appreciation occurs if $f_r m_w - f_w m_r < 0$, i.e. if the rise in the interest rate leads to a large substitution of bonds in favor of foreign assets combined with a low degree of substitution from money to bonds. Otherwise, the exchange rate will depreciate, as was the case when budget deficits are money financed.

(d) The short-run model above is now extended by allowing for price adjustments. The short-run effects of expansionary monetary policy (an open market operation) is a fall in the interest rate and a depreciated currency. We can then trace out the long-run effects starting with the short-run effects.

Assume that the economy is in full equilibrium initially, the three asset markets are in equilibrium, net foreign investment income is zero and the trade balance is zero such that the current account is zero. Normalize the exchange rate and the price level such that they are both equal to unity.

The graph below illustrates.



Given our assumptions above we know that at time t_0, $S_0 = P_0 = 1$ (point A) and the trade balance is zero. At time t_0 the government is conducting expansionary monetary policy and the interest rate falls whereas the exchange rate depreciates. In the graph, the exchange rate increases to S_1 (point C). As a consequence, the trade balance improves, there is an increase in exports and a fall in imports (point G). There is now a current account surplus, the economy accumulates foreign assets. There is excess supply of foreign bonds and the agents re–balance their portfolios by selling foreign bonds and buying domestic bonds. The exchange rate starts to appreciate in response, a movement from S_1 at point C towards point E. The appreciated currency leads to a decline in competitiveness such that the trade balance deteriorates, there is a movement from point G to point H. The increase in the money supply will raise the price level to the new long-run level P_1, the price level increases from point A to point E and B such that the long-run level is equal to P_1. At point E, the exchange rate is equal to the price level such that the real exchange rate is unity and equal to the initial real exchange rate. Trade balance is zero at point I. Current account surplus. In the long-run, there will be a trade balance deficit (point H). The long-run price level is P_1, the nominal exchange rate has appreciated to S_2, and the real exchange rate S/P is lower than initially.

The long-run effects of expansionary monetary policy are: a depreciated currency, a higher price level (P_1), a lower real interest rate, a trade balance deficit and a current account equal to zero.

2. This question relates the following learning objectives. Knowledge: Account for how

data releases affect exchange rate and summarize the empirical evidence. Skills: Master and apply microstructure based models to analyze price determination on the foreign exchange market.

The model stated in this question is a simple reformulation of a standard micro-based macro model. Equation (5) is UIP with a risk premium, equation (7) is the definition of the real exchange rate and equations (6) and (8) are Taylor-rules. Note that we have assumed that the domestic Taylor-rule is extended such that we assume that the monetary authority also condition the policy rate on the level of the real exchange rate.

(a) We should show that the dealer' expectation of the exchange rate in the next period is equal to equation (9). To do this we first lead the UIP relation in equation (5) one period and then we take the expectation using information available at time t. We then have

$$\mathbb{E}_t^D s_{t+1} = \mathbb{E}_t^D \left[\hat{r}_{t+1} - r_{t+1} - \delta_{t+1}\right] + \mathbb{E}_t^D s_{t+2}$$

Next, take the difference between the two Taylor-rules

$$\mathbb{E}_t^D \hat{r}_{t+1} - \mathbb{E}_t^D r_{t+1} = \gamma_\pi \left(\mathbb{E}_t^D \Delta\hat{p}_{t+2} - \mathbb{E}_t^D \Delta p_{t+2}\right) + \gamma_y \left(\mathbb{E}_t^D \hat{y}_{t+1} - \mathbb{E}_t^D y_{t+1}\right) - \gamma_\varepsilon \mathbb{E}_t^D \varepsilon_{t+1}$$

insert this into the UIP relation above and use the definition of the real exchange rate such that

$$\mathbb{E}_t^D s_{t+1} = \gamma_\pi \left(\mathbb{E}_t^D \Delta\hat{p}_{t+2} - \mathbb{E}_t^D \Delta p_{t+2}\right) + \gamma_y \left(\mathbb{E}_t^D \hat{y}_{t+1} - \mathbb{E}_t^D y_{t+1}\right) -$$

$$\gamma_\varepsilon \left(\mathbb{E}_t^D s_{t+1} + \mathbb{E}_t^D \hat{p}_{t+1} - \mathbb{E}_t^D p_{t+1}\right) - \mathbb{E}_t^D \delta_{t+1} + \mathbb{E}_t^D s_{t+2}$$

Simplify and divide by $1 + \gamma_\varepsilon$

$$\mathbb{E}_t^D s_{t+1} = \frac{1}{1+\gamma_\varepsilon}\left[\gamma_\pi \left(\mathbb{E}_t^D \Delta\hat{p}_{t+2} - \mathbb{E}_t^D \Delta p_{t+2}\right) + \gamma_y \left(\mathbb{E}_t^D \hat{y}_{t+1} - \mathbb{E}_t^D y_{t+1}\right)\right] -$$

$$\frac{\gamma_\varepsilon}{1+\gamma_\varepsilon}\left(\mathbb{E}_t^D \hat{p}_{t+1} - \mathbb{E}_t^D p_{t+1}\right) - \frac{1}{1+\gamma_\varepsilon}\mathbb{E}_t^D \delta_{t+1} + \frac{1}{1+\gamma_\varepsilon}\mathbb{E}_t^D s_{t+2}$$

Let $\rho = \frac{1}{1+\gamma_\varepsilon}$ and define the fundamentals f_{t+1} as

$$\mathbb{E}_t^D f_{t+1} = \gamma_\pi \left(\mathbb{E}_t^D \Delta\hat{p}_{t+2} - \mathbb{E}_t^D \Delta p_{t+2}\right) + \gamma_y \left(\mathbb{E}_t^D \hat{y}_{t+1} - \mathbb{E}_t^D y_{t+1}\right) - \frac{1-\rho}{\rho}\left(\mathbb{E}_t^D \hat{p}_{t+1} - \mathbb{E}_t^D p_{t+1}\right)$$

Then we have

$$\mathbb{E}_t^D s_{t+1} = \mathbb{E}_t^D \rho \left(f_{t+1} - \delta_{t+1}\right) + \mathbb{E}_t^D \rho s_{t+2}$$

and solve this difference equation

$$\mathbb{E}_t^D s_{t+1} = \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i \left(f_{t+i} - \delta_{t+i}\right) + \mathbb{E}_t^D \lim_{i\to\infty} \rho^i s_{t+1+i}$$

where we note that $\rho < 0$ implying that the limit term converges to zero. Then we have equation (9) in the question.

(b) We have now derived the dealers' expectation of the future exchange rate that we can insert into the UIP relation in equation (5) to obtain

$$s_t - (\hat{r}_t - r_t - \delta_t) = \mathbb{E}_t^D s_{t+1}$$

Simplify this

$$s_t = (\hat{r}_t - r_t) + \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i f_{t+i} - \mathbb{E}_t^D \sum_{i=0}^{\infty} \rho^i \delta_{t+i}$$

which is identical to equation (10)

(c) There are different scenarios we can think of here. For example, the Trump twitter could put pressure on the Fed to cut the interest rate. According to the portfolio balance model, we then would expect that the exchange rate depreciates. Alternatively, a lower domestic interest rate implies a future appreciation of the exchange rate according to UIP. Therefore, the exchange rate must depreciate immediately.

In terms of the micro-based macro model above, a cut in the domestic interest rate should have an immediate and contemporaneous effect through the first term in equation (10). But, the Fed did not respond immediately to the twitter so there is no change in the interest rate differential. The question then is how dealers' expectation about future fundamentals (output and inflation) and about future risk premia will be affected. A higher domestic inflation rate will imply a depreciation and if this depreciation also leads to increased exports and thus output, then the exchange rate must depreciate. The question then is how the risk premia is affected. One could imagine that investors perception of economic policy in the US becomes less clear, that is, twitter activity may tend to increase uncertainty about future economic policy and thus raise the risk relative to other countries. If that is the case, then δ should fall (δ measures the risk premia of the foreign economy) and the exchange rate should depreciate further. On the other hand, investors may view the presidents announcements as new direction of monetary policy reducing the uncertainty. If that is the case, then we could expect a rise in the risk premia and the exchange should appreciate.

The lesson is that we cannot find an unambiguous answer to the question how twitter announcements should affect the exchange rate, it all depends on how market participants view the new information. Is it credible or not? Does it announce a change in the economic policy and so on? Ultimately, this is an empirical question.